Application Note

Operational Amplifiers

Noise Calculations of Op-Amp Circuits

Abstract

This application note describes calculating amplifier circuit noise by using the noise parameters from the op-amp datasheet; for engineers new to the subject of op-amp noise, the document guides you in calculating the possible output noise for common amplifier circuits.

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Related Literature

For a full list of related documents, visit our website:

ISL28136, ISL28218 device pages

1. Introduction

Due to its random nature, noise is computed using root-mean-square (rms) values of noise voltages and noise currents. However, op-amp data sheets commonly specify noise voltage and current in the form of spectral densities. This application note provides the necessary equations to convert noise spectral densities into rms noise.

To distinguish between signal and noise quantities as well as between rms noise and spectral densities, this document uses the following nomenclature:

V_S = rms signal voltage

E_n = rms noise voltage

e_n = noise voltage spectral density

I_n = rms noise current

i_n = noise current spectral density

2. Inherent Circuit Noise

An op-amp circuit exhibits internal noise, randomly, because (1) the random generation and recombination of electron-hole pairs in semi-conductors and (2) the thermal agitation of electrons in resistors. From thermal agitation, each vibrating electron inside a resistor constitutes a minuscule current; these currents add up to a net current and, therefore, a net voltage. This voltage, although zero on average, is constantly fluctuating because of the random distribution of the instantaneous magnitudes and directions of the individual currents. As a result, each node voltage and branch current in a circuit is constantly fluctuating around its desired nominal value.

2.1 Signal-to-Noise Ratio (SNR)

Noise degrades the quality of a signal by posing a limit on the size of the signal that can be successfully detected. A measure of specifying the signal quality in the presence of noise is the signal-to-noise ratio (SNR). The SNR is commonly defined as the ratio of the rms signal voltage, to the rms noise voltage in dB:

(EQ. 1)

$$SNR = 20\log_{10}\left(\frac{V_s}{E_n}\right) = 20\log_{10}\left(\frac{\text{rms Signal voltage}}{\text{rms Noise voltage}}\right)$$

In sensor applications, voltage amplifiers amplify small sensor signals to a level where they can be processed by analog-to-digital converters (ADCs). In this case, you must know the SNR because it determines the required resolution of the ADC. Evidently, the lower the SNR value, the lower the ADC resolution, and the more difficult it is to rescue the useful signal from noise.

2.2 Noise Gain

In op-amp circuits, noise is commonly assigned to the non-inverting op-amp input, so for the inverting amplifier configuration, the noise gain differs from the signal gain; for the non-inverting amplifier, both gains are identical (Figure 1).

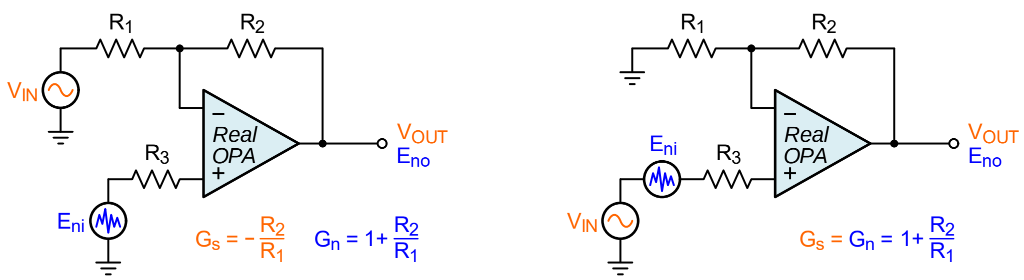

Figure 1. Noise and Signals Gains for the Inverting and Non-Inverting Amplifier

2.3 Noise Summation

An amplifier circuit contains multiple noise sources, generated by the op-amp and its surrounding resistors. Due to the random nature of noise, finding the total circuit input requires adding the individual noise sources quadratically:

(EQ. 2)

$$E_n = \sqrt{E_{n1}^2 + E_{n2}^2 + \ldots + E_{ni}^2}$$

and

(EQ. 3)

$$I_n = \sqrt{I_{n1}^2 + I_{n2}^2 + \ldots + I_{ni}^2}$$

3. Receiver Functional Principle

Op-amp noise is modeled by a noiseless op-amp equipped with two equivalent noise sources at the input: a voltage source with spectral density e_n and a current source with density (Figure 2). Op-amp noise is a mixture of 1/f noise and broadband or white noise. The spectral densities of 1/f noise, denoted as e_{nf} and i_{nf}, decline with frequency at a rate of 0.5dec/dec. The spectral densities of white noise, denoted as e_{nw} and i_{nw}, remain constant versus frequency (Figure 3).

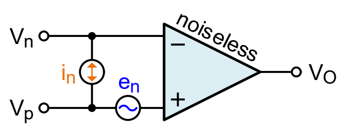

Figure 2. Op-Amp Noise Model

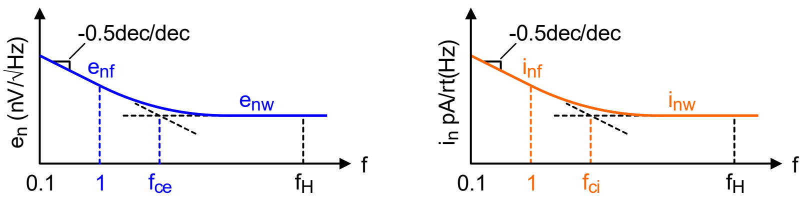

Figure 3. Voltage and Current Noise Spectral Densities

To find the total rms input noise of an op-amp for a given bandwidth, the spectral densities in Figure 3 must be converted into the rms noise voltage and current using Equation 4 and Equation 5:

(EQ. 4)

$$E_n = e_{nw}\sqrt{f_{ce} \cdot \ln\frac{f_H}{f_L} + n \cdot f_H - f_L}$$

and

(EQ. 5)

$$I_n = i_{nw}\sqrt{f_{ci} \cdot \ln\frac{f_H}{f_L} + n \cdot f_H - f_L}$$

where $n \cdot f_H - f_L$ is the white noise equivalent bandwidth (see White Noise Equivalent Bandwidth (NEB)), and f_{ce} and f_{ci} are the corner frequencies, where the op-amp noise transitions from the 1/f noise region to the white noise region. The corner frequencies can be calculated with:

(EQ. 6)

$$f_{ce} = f_x\left[\left(\frac{e_{nf(fx)}}{e_{nw}}\right)^2 - 1\right]$$

and

(EQ. 7)

$$f_{ci} = f_x\left[\left(\frac{i_{nf(fx)}}{i_{nw}}\right)^2 - 1\right]$$

where the $e_{nf(fx)}$ and $i_{nf(fx)}$ are the 1/f spectral densities at an arbitrary frequency, fx.

Note: The rms noise current, I_n, is converted into an rms noise voltage, E_n, when flowing through a resistor:

(EQ. 8) $$E_n = R \cdot I_n = R \cdot i_{nw} \sqrt{f_{ci} \cdot \ln\frac{f_H}{f_L} + n \cdot f_H - f_L}$$

3.1 White Noise Equivalent Bandwidth (NEB)

An interesting phenomenon occurs when running white noise through a first order low-pass filter with f_H as its -3dB frequency. In this case, the noise behaves as if filtered by a brick-wall filter with a higher cutoff frequency of $f_c = 1.57 f_H$ (Figure 4).

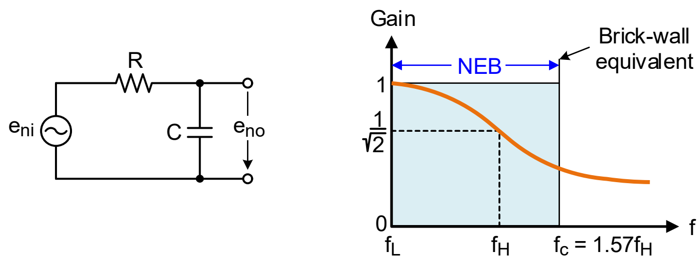

Figure 4. Noise Equivalent Bandwidth (NEB) for 1st Order Low-Pass Filter

This increased noise bandwidth is known as the white noise equivalent bandwidth (NEB). With increasing filter order, f_c decreases and approaches f_H, because the steeper roll-off of higher filter orders approaches the brick-wall equivalent. Mathematically, the NEB is expressed through:

(EQ. 9) $$NEB = f_c - f_L = n \cdot f_H - f_L$$

where n is the brick-wall factor for a given filter order. The higher the filter order, the closer the value of n approaches 1. Figure 5 lists the values of n for the first five orders of passive low-pass filters.

Filter Order	n
1	1.57
2	1.22
3	1.16
4	1.13
5	1.12

Figure 5. Values of n for Higher Order Filters

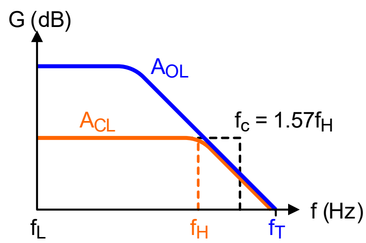

Figure 6. NEB of Amplifier with Resistive Feedback

For amplifiers with purely resistive feedback, the closed-loop gain represents a first order low-pass with a -3dB bandwidth, f_H. (Figure 6). These amplifiers pass white noise with a cutoff frequency of $f_c = 1.57 \cdot f_H$.

Note: Find f_H by taking the unity-gain bandwidth, f_T, from the op-amp data sheet and dividing it by the noise-gain, $1/\beta$:

(EQ. 10) $$f_H = f_T / (1/\beta)$$

4. Thermal Noise in Resistors

Resistor noise is thermal noise due to the random thermal motion of electrons. It is present in standard resistors as well as in the stray resistances of practical inductors and capacitors. Resistor noise can be modeled either by a noise voltage of spectral density, e_R, in series, or by a noise current of spectral density, i_R, in parallel with a noiseless resistor (Figure 7).

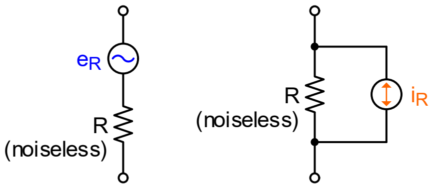

Figure 7. Thermal (Resistor) Noise Models

The spectral densities of thermal noise are defined through:

(EQ. 11)
$$e_R = \sqrt{4kTR}$$

and

(EQ. 12)
$$i_R = \sqrt{\frac{4kT}{R}}$$

where $k = 1.38 \cdot 10^{-23}$ J/K is Boltzmann's constant, T is the absolute temperature in kelvins, and R is the resistance in ohms.

Equation 11 and Equation 12 show that the spectral densities of thermal noise, e_R and i_R, are constant with frequency, therefore indicating that thermal noise is a type of white noise with its upper bandwidth limit at $f_c = n \cdot f_H$.

Thermal noise is commonly represented as an rms noise voltage. Whether using the e_R or i_R spectral density, the resulting rms noise voltage is always:

(EQ. 13)
$$E_R = \sqrt{4kTR(n \cdot f_H - f_L)}$$

Note: The equation holds true for resistors connected in series or parallel. For example, if two resistors are connected in series, $R = R_1 + R_2$, if they are parallel connected, $R = R_1 \,||\, R_2$.

If resistors are connected in parallel, use the i_R-model. In this case, each noise current flows through the parallel resistance $R_P = R_1 \,||\, R_2$, therefore producing the individual noise voltages.

5. Total RMS Input Noise, Eni

When calculating the total rms input noise of a voltage amplifier, E_{ni}, convert the initial circuit schematic (Figure 8) into a noise equivalent circuit diagram (Figure 9) and apply the equations from the previous sections.

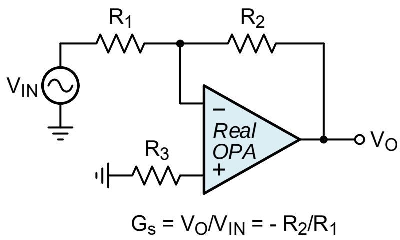

Figure 8. Inverting Amplifier

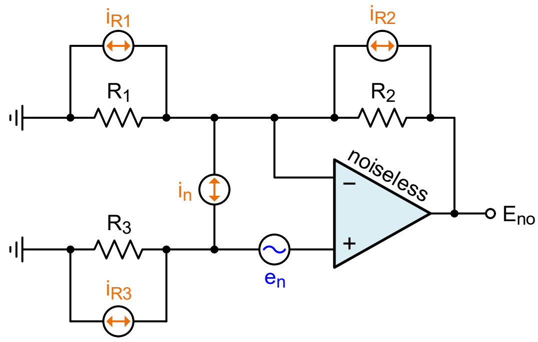

Figure 9. Spectral Noise Equivalent Circuit

Starting with the thermal noise calculation: noise-wise, R_1 and R_2 are parallel connected, therefore generating an rms noise voltage:

(EQ. 14)
$$E_{Rp} = \sqrt{4kT(1.57f_H - f_L) \cdot R_1 \| R_2}$$

The other thermal noise source is R_3, with the following rms value:

(EQ. 15)
$$E_{R3} = \sqrt{4kT(1.57f_H - f_L) \cdot R_3}$$

Next, calculate the rms voltages that are due to op-amp current noise. At the negative input, this voltage, denoted as E_{nn}, is the product of the rms noise current, I_n, and the parallel resistance R_1 || R_2:

(EQ. 16)
$$E_{nn} = R_1 \| R_2 \cdot i_{nw} \sqrt{f_{ci} \cdot \ln(f_H/f_L) + 1.57f_H - f_L}$$

At the positive input, the rms noise voltage, denoted as E_{np}, is the product of I_n, and R_3:

(EQ. 17)
$$E_{np} = R_3 \cdot i_{nw} \sqrt{f_{ci} \cdot \ln(f_H/f_L) + 1.57f_H - f_L}$$

Finally, calculate the rms input noise of the op-amp itself with the following:

(EQ. 18)
$$E_n = e_{nw} \sqrt{f_{ce} \cdot \ln(f_H/f_L) + n \cdot f_H - f_L}$$

Summing up all rms noise voltages above in rms fashion yields the total input noise, E_{ni}, of the circuit:

(EQ. 19)
$$E_{ni} = \sqrt{E_{Rp}^2 + E_{R3}^2 + E_{nn}^2 + E_{np}^2 + E_n^2}$$

Writing Equation 19 in detailed form results in a huge expression:

$$E_{ni} = \sqrt{\begin{aligned}&4kT(1.57f_H - f_L)\cdot R_1 \| R_2 + 4kT(1.57f_H - f_L)\cdot R_3 + (R_1 \| R_2 \cdot i_{nw})^2 \cdot [f_{ci}\cdot \ln(f_H/f_L) + 1.57f_H - f_L] \\ &+ (R_3 \cdot i_{nw})^2 \cdot [f_{ci}\cdot \ln(f_H/f_L) + 1.57f_H - f_L] + e_{nw}{}^2 \cdot [f_{ce}\cdot \ln(f_H/f_L) + n\cdot f_H - f_L]\end{aligned}}$$

However, you can simplify the above equation by making the following realistic assumptions that apply to the majority of amplifier circuits:

$f_H > 10 \cdot f_{ce}$: The upper bandwidth limit is more than 10 times the corner frequency of 1/f and white noise, which makes the term neglectable, therefore leaving only $n \cdot f_H - f_L$ under the square root.

$f_H >> f_L$: The upper bandwidth limit is much larger than the lower bandwidth limit of 0.1Hz typically, which makes f_L neglectable, therefore reducing the $(n \cdot f_H - f_L)$ term to $n \cdot f_H$.

n = 1.57: Due to a 1st order low-pass response, either because of purely resistive feedback, or a simple output filter.

$R_3 = R_1 \| R_2 = R_P$: To minimize the offset due to bias current.

Applying the above assumptions and after factoring out and collecting terms, results in the following:

(EQ. 20)
$$E_{ni} = \sqrt{1.57f_H} \cdot \sqrt{8kTR_P + 2R_P{}^2 i_{nw}{}^2 + e_{nw}{}^2}$$

Next, multiplying E_{ni} with the noise gain yields the total rms output noise, E_{no}, of the circuit:

(EQ. 21)
$$E_{no} = E_{ni} \cdot (1 + R_2/R_1)$$

6. Calculation Example 1

Calculate the output signal-to-noise ratio of an inverting amplifier using the op-amp ISL28136.

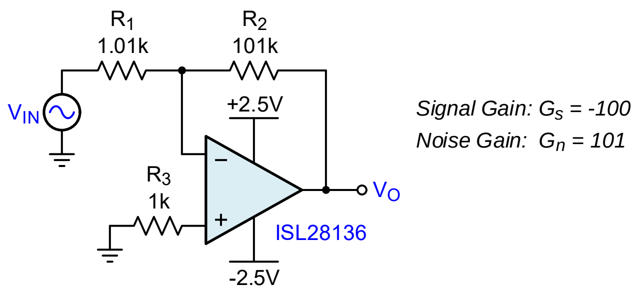

Figure 10. Inverting Amplifier with ISL28136

From the AC specification in the op-amp datasheet we take the following data:

GBW = 5MHz, e_{nw} = 15nV/√Hz, i_{nw} = 0.35pA/√Hz. The corner frequencies of the voltage and current noise densities are calculated with Equation 6 and Equation 7, yielding f_{ce} = 30Hz and f_{ci} = 521Hz (see Figure 11):

$$f_{ce} = 10 \cdot \left[\left(\frac{30}{15} \right)^2 - 1 \right] = 30\,\text{Hz}$$

$$f_{ci} = 1 \cdot \left[\left(\frac{8}{0.35} \right)^2 - 1 \right] = 521\,\text{Hz}$$

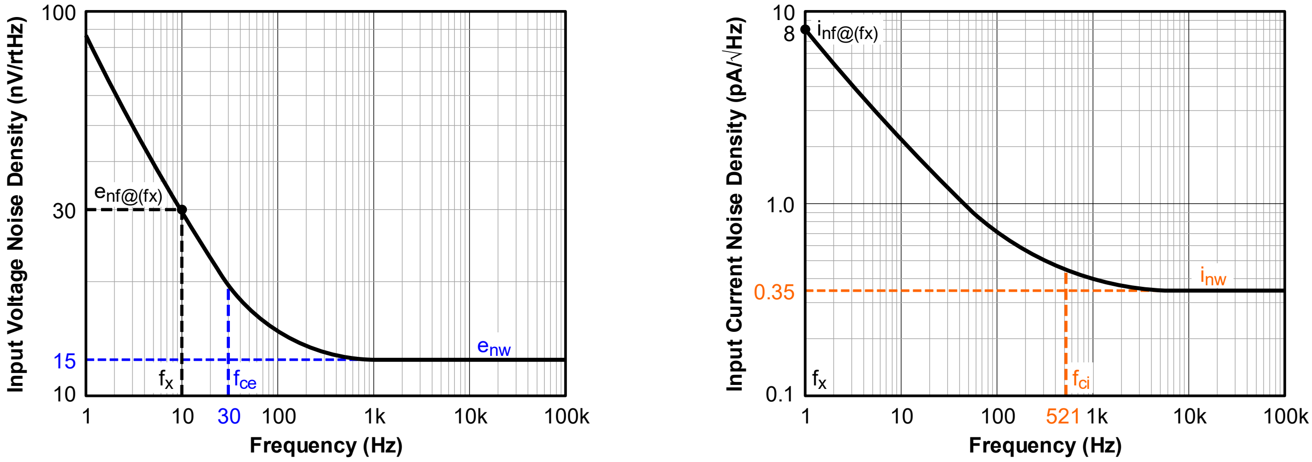

Figure 11. Inverting Amplifier with ISL28136

Table 1. Detailed Calculations

Bandwidth Limits	
$f_L = 0.1\text{Hz}$	Assumed lower bandwidth limit
$f_H = \frac{GBW}{G_n} = \frac{5\text{MHz}}{101} = 49.5\text{kHz}$	Upper bandwidth limit for 1/f noise
NEB = 1.57 · $f_H - f_L$ = 77.7kHz	White noise equivalent bandwidth
Thermal Noise Contribution	
$E_{Rp} = \sqrt{4kT \cdot NEB \cdot R_1 \Vert R_2}$ $E_{Rp} = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 298K \cdot 77.7\text{kHz} \cdot 1\text{k}\Omega} = 1.13\mu\text{V}$	Thermal noise of R_1 and R_2 at T = 298 Kelvin (25°C) using Equation 13
$E_{R3} = E_{Rp} = 1.13\mu\text{V}$	Thermal noise of R_3 is the same as in R_1 || R_2 due to R_3 = R_1 || R_2.
Noise Voltage due to Op-amp Current Noise	
$E_{nn} = R_P \cdot i_{nw} \sqrt{f_{ci} \cdot \ln(f_H/f_L) + NEB}$ $E_{nn} = 1\text{k}\Omega \cdot 0.35 \frac{\text{pA}}{\sqrt{\text{Hz}}} \cdot \sqrt{521\text{Hz} \cdot \ln\left(\frac{49.5\text{kHz}}{0.1\text{Hz}}\right) + 77.7\text{kHz}} = 102\text{nV}$	Voltage noise due to op-amp noise current flowing through R_1 || R_2 using Equation 8
$E_{np} = E_{nn} = 102\text{nV}$	Voltage noise due to op-amp noise current flowing through R3 is the same as in R_1 || R_2 due to R_3 = R_1 || R_2.
Op-amp Input Noise Voltage	
$E_n = e_{nw} \sqrt{f_{ce} \cdot \ln(f_H/f_L) + NEB}$ $E_n = 15 \frac{\text{nV}}{\sqrt{\text{Hz}}} \cdot \sqrt{30\text{Hz} \cdot \ln\left(\frac{49.5\text{kHz}}{0.1\text{Hz}}\right) + 77.7\text{kHz}} = 4.192\mu\text{V}$	RMS value of op-amp voltage noise using Equation 4 and Equation 5

Table 1. Detailed Calculations (Continued)

Total Input Noise per Equation 19	
$E_{ni} = \sqrt{E_{Rp}{}^2 + E_{R3}{}^2 + E_{nn}{}^2 + E_{np}{}^2 + E_n{}^2} = \sqrt{2E_{Rp}{}^2 + 2E_{nn}{}^2 + E_n{}^2}$ $E_{ni} = \sqrt{(1.599\mu V)^2 + (0.144\mu V)^2 + (4.192\mu V)^2} = 4.489\mu V$	Total RMS Input Noise if Equation 19 is applied with $R_3 = R_1 \parallel R_2$.
Total Input Noise using simplified Equation 20	
$E_{ni} = \sqrt{NEB \cdot (8kTR_P + 2R_P{}^2 i_{nw}{}^2 + e_{nw}{}^2)}$ $E_{ni} = \sqrt{77.7kHz \cdot [32.9 \cdot 10^{-18} + 245 \cdot 10^{-21} + 225 \cdot 10^{-18}]} = 4.479\mu V$	Total RMS Input Noise if simplified Equation 20 is applied. Error is only 0.23%.
Total Output Noise	
$E_{no} = E_{ni} \cdot G_n = 4.489\mu V \cdot 101 = 453.4\mu V$	Total RMS Output Noise
Output SNR	
$SNR = 20 \cdot \log_{10}\left(\frac{V_{O-pp}}{2\sqrt{2}E_{no}}\right) = 20 \cdot \log_{10}\left(\frac{4.6V}{2\sqrt{2} \cdot 453.4\mu V}\right) = 71dB$	Output SNR for Output FSR = $4V_{PP}$

The dominant noise source in the highlighted E_{ni} term is the 4.2µV, which is caused by the high input noise voltage, e_{nw}, of the op-amp. A rule-of-thumb states that if a dominant noise source is three-times higher than the other noise sources in the system, the other noise sources can be neglected, and noise reduction efforts should focus on the dominant noise source. In other words, to get a higher SNR, you need an op-amp with lower voltage noise. When this is achieved, further noise reduction can be obtained by reducing the resistor values.

7. Calculation Example 2

To reduce the output noise from the previous circuit example without narrowing the overall bandwidth, select the dual low-noise op-amp ISL28218 for a two-stage amplifier design. This allows you to use symmetrical lower gains, and each stage has a gain of 10 and also lower resistor values.

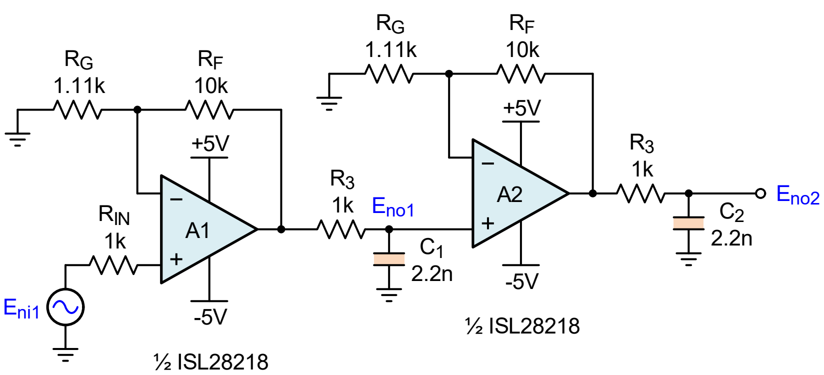

Figure 12. Two-Stage Amplifier with ISL28218

The key parameters of the ISL28218 are: GBW = 3.2MHz, e_{nw} = 5.5nV/√Hz, i_{nw} = 0.38pA/√Hz.

The closed-loop bandwidth of each stage is:

(EQ. 22)

$$f_{Acl} = \frac{GBW}{G_n} = \frac{3.2MHz}{10} = 320kHz$$

The external low-pass filters however, provide an earlier cutoff at:

(EQ. 23) $$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 1k\Omega \cdot 2.2nF} = 72.4kHz = f_H$$

And therefore, determine the equivalent noise bandwidth, NEB.
Note: The NEB of the two gain stages differ, because the noise of the first stage experiences two low-pass functions (of Filter 1 and Filter 2), while the second stage sees only one low-pass. The NEB is therefore:

(EQ. 24) $$NEB_1 = 1.22 \cdot f_H = 1.22 \cdot 72.4kHz = 88.3kHz$$

The 2nd stage faces only one low-pass function, therefore making the NEB:

(EQ. 25) $$NEB_2 = 1.57 \cdot f_H = 1.57 \cdot 72.4kHz = 113.6kHz$$

To reduce the mathematical effort, apply Equation 20 using to calculate the rms input noise for each stage under the condition that:

(EQ. 26) $$E_{ni1} = \sqrt{NEB_1 \cdot \left(8kTR_P + 2R_P{}^2 i_{nw}{}^2 + e_{nw}{}^2\right)} = 2.367\mu V$$

(EQ. 27) $$E_{ni2} = \sqrt{NEB_2 \cdot \left(8kTR_P + 2R_P{}^2 i_{nw}{}^2 + e_{nw}{}^2\right)} = 2.685\mu V$$

Each input noise contributes individually to the overall output noise. We denote E_{no21} as the output noise due to E_{ni1}, and E_{no22}, as the output noise due to E_{ni2}:

(EQ. 28) $$E_{no21} = E_{ni1} \cdot G_{n1} \cdot G_{n2} = 2.367\mu V \cdot 10 \cdot 10 = 236.7\mu V$$

(EQ. 29) $$E_{no2} = E_{ni2} \cdot G_{n2} = 2.685\mu V \cdot 10 = 26.85\mu V$$

The total rms output noise is:

(EQ. 30) $$E_{no} = \sqrt{E_{no1}{}^2 + E_{no1}{}^2} = 238.2\mu V$$

While maintaining the same signal bandwidth, of about 48kHz, we have reduced the rms output noise from 453µV down to 238µV. Assuming the same $4V_{pp}$ full scale voltage range at the output, the new SNR is:

(EQ. 31) $$SNR = 20 \cdot \log_{10}\left(\frac{V_{O-pp}}{2\sqrt{2}E_{no}}\right) = 20 \cdot \log_{10}\left(\frac{4.6V}{2\sqrt{2} \cdot 238.2\mu V}\right) = 76.7dB$$

which is an improvement of nearly 6dB, or twice the resolution.

8. Conclusion

Noise calculations can be tedious. Applying correct, realistic assumptions drastically reduces the mathematical effort. While this application note is not a replacement for the vast amount of technical documentation on noise, this document helps engineers new to the subject of op-amp noise quickly assess the possible output noise for common amplifier circuits.

9. Revision History

Rev.	Date	Description
1.00	Aug.4.20	Initial release

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