Operational Amplifier Basics

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Abstract

This tech brief provides a short and simple introduction to operational amplifiers, their functionality, and some basic applications concepts. It is intended for people new to the subject.

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1. Op-Amp Function

An op-amp is a signal amplifier with high DC-gain in the range of 100000 to 10000000. The amplifier has three signal pins:

- A positive or non-inverting signal input (IN+)
- A negative or inverting signal input (IN-)
- A signal output (OUT)

Throughout this tech brief, assume the op-amp to be ideal, meaning:

- Both inputs have infinite input impedance and hence, no input currents flowing into them
- The output impedance is zero and thus, providing infinite current drive

To distinguish between the applied input voltage or voltages and the actual voltage potentials residing at the op-amp input terminals, the latter will be indicated by lowercase letters and indices.

Figure 1 shows the internal signal flow of an op-amp. The input signals at the positive and negative inputs combine at the internal summing point, Σ. Because the signal at IN- is internally inverted, v_n is subtracted from v_p, resulting in the difference voltage $v_d = v_p - v_n$. This voltage is then amplified by the very high amplifier gain to yield the output voltage $V_O = v_d \cdot A$.

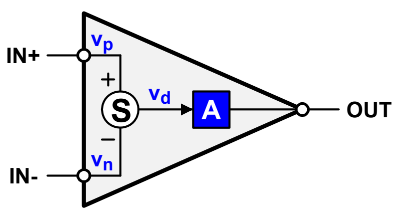

Figure 1. Operational Amplifier

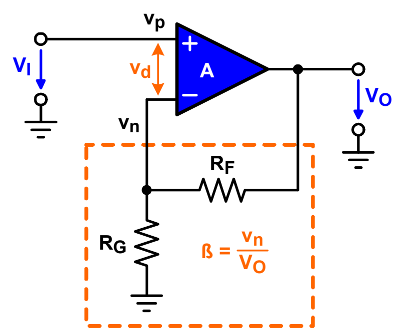

Figure 2. Amplifier Circuit

Op-amps are never used as standalone devices but require an additional voltage divider that couples a fraction of the output voltage back into the negative input (Figure 2). The attenuation of the voltage divider is known as the feedback factor, β. This factor is important as it determines the overall gain of the amplifier circuit.

This can be proven by converting the circuit in Figure 2 into a signal flow model, using the op-amp's internal structure shown in Figure 1. The resulting model is that of a non-inverting amplifier circuit, because the input voltage is applied to the non-inverting input (Figure 3 on page 2).

The circuit gain, also known as transfer function, is the ratio of output parameter to input parameter, in this case V_O/V_I.

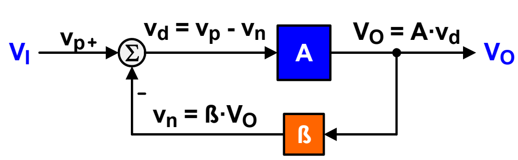

Figure 3. Model of Non-Inverting Amplifier Circuit

To find the transfer function, we establish the equation for each one of the three signal nodes.

(EQ. 1) $$V_O = A \cdot v_d$$

(EQ. 2) $$v_d = V_I - v_n$$

(EQ. 3) $$v_n = \beta \cdot V_O$$

Inserting Equations 2 and 3 into Equation 1 gives VO = A and solving for V_O/V_I results:

(EQ. 4) $$\frac{V_O}{V_I} = \frac{A}{1 + A \cdot \beta}$$

Dividing the nominator and denominator through Aβ yields:

(EQ. 5) $$\frac{V_O}{V_I} = \frac{1}{\beta} \cdot \frac{1}{1 + \frac{1}{A \cdot \beta}}$$

where

$\frac{1}{\beta}$ presents the ideal circuit gain

and

$\frac{1}{1 + \frac{1}{A \cdot \beta}}$ the gain accuracy

Let's assume two amplifier circuits, both of which have the same feedback factor of β = 0.1, but different op-amp gains of A1 = 10000 and A2 = 100000. Per Equation 5 the resulting circuit gains G1 and G2 are:

$$G_1 = \frac{V_O}{V_I} = \frac{1}{\beta} \cdot \frac{1}{1 + \frac{1}{A_1 \cdot \beta}} = \frac{1}{0.1} \cdot \frac{1}{1 + \frac{1}{10^4 \cdot 0.1}} = 10 \cdot 0.999 = 9.990$$

$$G_2 = \frac{V_O}{V_I} = \frac{1}{\beta} \cdot \frac{1}{1 + \frac{1}{A_2 \cdot \beta}} = \frac{1}{0.1} \cdot \frac{1}{1 + \frac{1}{10^5 \cdot 0.1}} = 10 \cdot 0.9999 = 9.999$$

While the ideal gain is the same in both cases, the gain accuracy is higher in the circuit with the higher op-amp gain.

In summary:

(1) The feedback factor, β, defines the ideal circuit gain**.**

(2) The op-amp gain, A, determines how close the real circuit gain approximates the ideal value.

2. Steady State Voltages

When applying an input voltage step (V_I) to the circuit in Figure 4, the amplifier output (V_O) increases steadily and so does the output of the voltage divider (v_n), although at a slower rate.

Both voltages continue to rise until v_n is so close to V_I that their difference (v_d) is equal to V_O/A. At this moment, an equilibrium between the op-amp input and output is reached. This is known as the steady state, because no further voltage movements occur unless the input voltage is changed again.

Note that v_n will never reach V_I, otherwise v_d would be 0V, which would make the output = 0V. Thus, there is always a minimum differential input required to maintain V_O at the correct, amplified level. This minimum v_d can be calculated by dividing the output voltage through the op-amp gain: $v_d = V_O/A$. For example, an op-amp with a gain of 100000 requires a minimum differential input of 50µV to produce a 5V output.

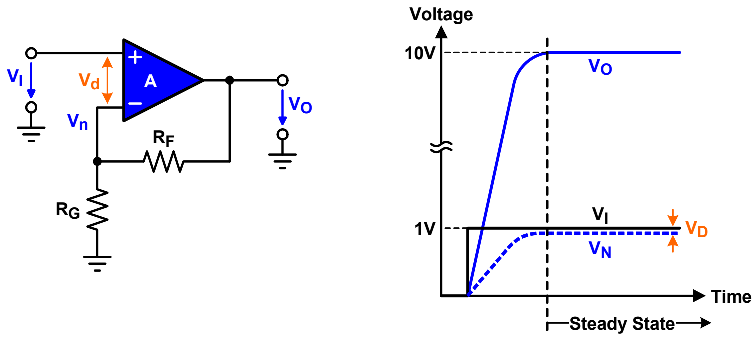

Figure 4. Steady State Voltages of a Non-Inverting Amplifier

In comparison to the amplifiers input and output voltage levels however, v_d is so small that it is assumed to be 0V. Therefore, when evaluating the transfer function of an amplifier circuit, start with this single, most important condition:

$$v_p = v_n$$

3. The Non-Inverting Amplifier

While the previous sections used the non-inverting amplifier to outline some basic op-amp concepts, this section solely focuses on deriving its output voltage equation for practice purposes. **Note that the signal grounds for input and output voltages have been omitted for clarity.**

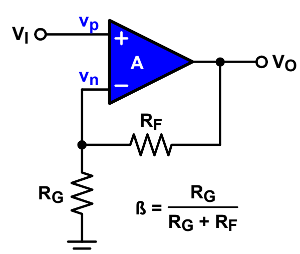

Figure 5. Non-Inverting Amplifier

Starting with the fundamental condition:

(EQ. 6) $$v_p = v_n$$

Each voltage at the input terminals is defined. In the case of v_p, this is easy because the input voltage is also the voltage at the positive input:

(EQ. 7) $$v_p = V_1$$

With regards to v_n, this voltage is the output of the R_F, R_G voltage divider with V_O as the driving source. Thus:

(EQ. 8) $$v_n = V_O \cdot \frac{R_G}{R_G + R_F}$$

Because of Equation 6, set Equation 7 equal to Equation 8

(EQ. 9) $$V_I = V_O \cdot \frac{R_G}{R_G + R_F}$$

Then solving for V_O gives:

$$V_O = V_I \cdot \frac{R_G + R_F}{R_G} = V_I \cdot \left(1 + \frac{R_F}{R_G}\right) \qquad \text{(EQ. 10)}$$

The output voltage of the non-inverting amplifier in Figure 5 is therefore given with

$$V_O = V_I \cdot \left(1 + \frac{R_F}{R_G}\right) \qquad \text{(EQ. 11)}$$

4. The Voltage Follower

The voltage follower is a non-inverting amplifier with a gain of 1, also known as unity-gain. Because $v_p = v_n$ and $V_I = v_p$ and $V_O = v_n$, it follows that $V_O = V_I$. Voltage followers are used as signal buffers or impedance converters, as they offer high input impedance to a driving signal source while providing low output impedance to signal loads.

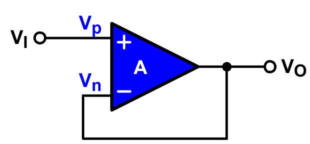

Figure 6. Voltage Follower

5. The Inverting Amplifier

The inverting amplifier seems a bit trickier because the input voltage is now applied to one end of the voltage divider to reach the negative input, while the positive input is grounded or 0V. Nevertheless, the $v_p = v_n$ condition still holds true.

Again, starting with

$$v_p = v_n \qquad \text{(EQ. 12)}$$

v_p is defined with 0V because the positive input is directly connected to signal ground.

$$v_p = 0 \qquad \text{(EQ. 13)}$$

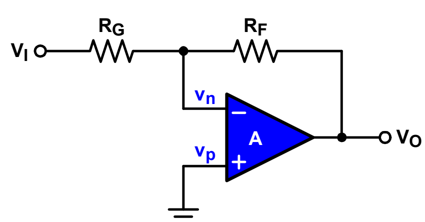

Figure 7. Inverting Amplifier

To determine v_n, observe that v_n is the summing point of a voltage divider that is driven by two voltage sources, V_O and V_I. Thus, when establishing the voltage divider ratio for V_I, assume $V_O = 0V$ and conversely, when establishing the voltage diver ratio for V_O, assume $V_I = 0V$. The sum of the output voltages of the two voltage diver calculations then gives v_n (Figure 8 on page 6).

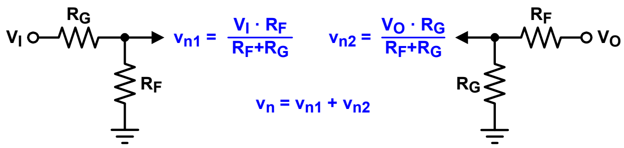

Figure 8. Determining v_n

(EQ. 14) $$v_n = V_I \cdot \frac{R_F}{R_F + R_G} + V_O \cdot \frac{R_G}{R_F + R_G}$$

According to Equations 12 and 13, $v_n = 0$, which makes the V_O term equaling the V_I term:

(EQ. 15) $$V_O \cdot \frac{R_G}{R_F + R_G} = -V_I \cdot \frac{R_F}{R_F + R_G}$$

Solving for V_O gives:

(EQ. 16) $$V_O = V_I \cdot \left(-\frac{R_F}{R_F + R_G}\right) \cdot \frac{R_F + R_G}{R_G} = -V_I \cdot \frac{R_F}{R_G}$$

The output voltage of the inverting amplifier in Figure 7 on page 5 is given with

$$V_O = -V_I \cdot \frac{R_F}{R_G}$$

The negative sign indicates the signal inversion or a 180° phase shift from input to output.

6. The Inverting Voltage Adder

Extending the number of inputs of an inverting amplifier leads to the inverting voltage adder in Figure 9.

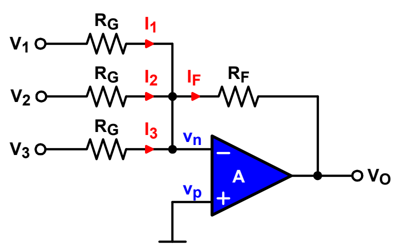

Figure 9. Inverting Voltage Adder

To define v_n, we apply Kirchhoff's Current Law (KCL) which states that the currents flowing into a node equal the currents flowing out of it, in this case $I_1+I_2+I_3 = I_F$. We then describe each current through its voltage/resistor ratio:

(EQ. 17) $$\frac{V_1 - v_n}{R_G} + \frac{V_2 - v_n}{R_G} + \frac{V_3 - v_n}{R_G} = \frac{v_n - V_O}{R_F}$$

With $v_n = v_p = 0$, Equation 17 simplifies to

$$\frac{V_1}{R_G} + \frac{V_2}{R_G} + \frac{V_3}{R_G} = \frac{-V_O}{R_F}$$

and solving for V_O gives:

$$V_O = -(V_1 + V_2 + V_3) \cdot \frac{R_F}{R_G} \qquad \text{(EQ. 18)}$$

7. The Non-Inverting Voltage Adder

This circuit uses the non-inverting amplifier configuration as its basis. The input circuit below is a voltage adder with an average voltage of:

$$v_p = \frac{V_1 + V_2 + V_3}{3} \qquad \text{(EQ. 19)}$$

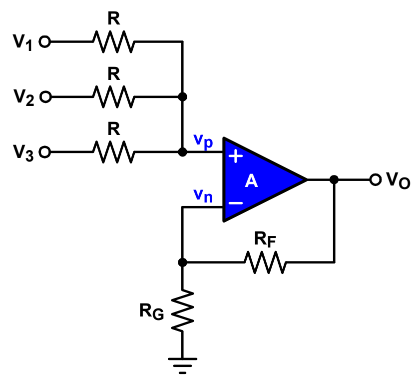

Figure 10. Non-Inverting Voltage Adder

With

$$v_n = V_O \cdot \frac{R_G}{R_G + R_F}$$

and making $v_p = v_n$ gives:

$$\frac{V_1 + V_2 + V_3}{3} = V_O \cdot \frac{R_G}{R_G + R_F} \qquad \text{(EQ. 20)}$$

Then solving for V_O results:

$$V_O = \frac{V_1 + V_2 + V_3}{3} \cdot \left(1 + \frac{R_F}{R_G}\right) \qquad \text{(EQ. 21)}$$

8. The Difference Amplifier or Voltage Subtractor

The difference amplifier combines the functions of a non-inverting amplifier with input V_1 and an inverting amplifier with input V_2.

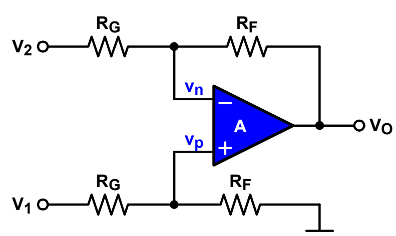

Figure 11. Difference Amplifier or Voltage Subtractor

The voltages at the op-amp input terminals are

$$v_p = V_I \cdot \frac{R_F}{R_F + R_G}$$

and

$$v_n = V_2 \cdot \frac{R_F}{R_F + R_G} + V_O \cdot \frac{R_G}{R_G + R_F}$$

Setting both terms equal and solving for V_O yields:

(EQ. 22) $$V_O = (V_1 - V_2) \cdot \frac{R_F}{R_G}$$

9. The Integrator

This circuit integrates the input voltage over time. To find the output equation we recall the capacitor's law,

$$I_C = C \cdot \frac{dV_C}{dt}$$

meaning the charge current, I_C, equals the product of capacitance, C, and the voltage change across the capacitor, dV_C, over the time interval, dt.

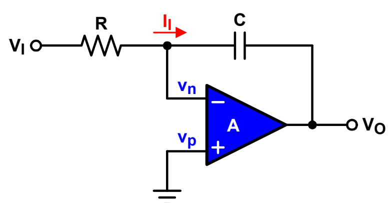

Figure 12. Integrating Amplifier

Now, V_C is the difference between v_n and V_O: $V_C = v_n - V_O$. Because no current flows into the op-amp, the charge current is also the input current I_I, which is defined by

$$I_I = \frac{(V_I - v_n)}{R}$$

Thus, the capacitor's law can be rewritten in the form of

$$\frac{(V_I - v_n)}{R} = C \cdot \frac{d(v_n - V_O)}{dt}$$

With $v_n = v_p = 0V$, this equation simplifies to

$$\frac{V_I}{R} = C \cdot \frac{(-dV_O)}{dt}$$

Then solving for dV_O gives

$$dV_O = -V_I \cdot \frac{dt}{(R \cdot C)}$$

and integrating on both sides results:

(EQ. 23) $$V_O(t) = \frac{-1}{RC}\int_0^t V_I(t)dt + V_O(0)$$

The op-amp integrator is used in many electronic applications, such as function generators to produce triangle and saw-tooth waveforms, as well as in active filters, A-to-D converters, and analog control loops.

10. The Differentiator

The analysis of the differentiator is similar to the one of the integrator. Because the current through the capacitor and the resistor is the same, it follows that

$$C \cdot \frac{d(V_I - v_n)}{dt} = \frac{(v_n - V_O)}{R}$$

and with $v_n = 0$,

$$C \cdot \frac{dV_I}{dt} = \frac{-V_O}{R}$$

Then, solving for V_O gives:

(EQ. 24) $$V_O = -RC \cdot \frac{dV_I(t)}{dt}$$

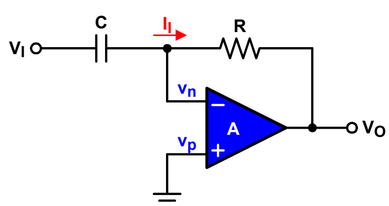

Figure 13. Differentiating Amplifier

The output is proportional to the derivative of the input with RC providing a constant of proportionality.

11. Conclusion

The basic op-amp circuits in this tech brief show that by interconnecting passive component networks around the op-amp, it can be configured for a variety of operations, such as multiplying by a constant, adding, subtracting, integrating, and differentiating. This explains why this amplifier is called operational.

12. Revision History

Rev.	Date	Description
0.00	May 1, 2018	Initial release

RENESAS

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