

Operational Amplifiers

Closed-Loop Gain Error

Abstract

Engineers new to operational amplifier (op-amp) circuit design often attribute amplifier gain error purely to resistor tolerances, while ignoring the gain error caused by the op-amp. This can prove detrimental, as the gain error of the op-amp can be higher than the one due to resistor tolerances. Also, op-amp gain error is frequency dependent, which can cause signal distortions when amplifying input signals containing multiple frequency components.

This application note explains the impact of op-amp open-loop gain on the closed-loop gain error and derives equations for the following:

- Calculating the gain error for a given op-amp at a specific frequency
- Identifying the maximum signal frequency that can be amplified within a specified gain error
- Determining the minimum gain bandwidth of an op-amp, required to amplify a given signal frequency within a specified gain error.

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1. Closed-Loop Gain Error as a Function of Open-Loop Gain for the Non-Inverting Amplifier

The gain error is defined as the deviation of the actual closed-loop gain of the amplifier from its ideal value. For example, the closed-loop gain, A_{CL} , or transfer function, V_O/V_I , of the non-inverting amplifier shown in [Figure 1](#) is:

$$(EQ. 1) \quad A_{CL} = \frac{1}{\beta} \cdot k_{Acc} \quad \text{with} \quad \frac{1}{\beta} = 1 + \frac{R_F}{R_G} \quad \text{and} \quad k_{Acc} = \frac{1}{1 + \frac{1}{A_{OL}\beta}}$$

Here, β is the feedback factor of the amplifier, and $1/\beta$ is the ideal closed-loop gain of the non-inverting amplifier for $A_{OL} = \infty$. The second factor, k_{Acc} , is the gain accuracy, which determines how close the actual closed-loop gain approaches the ideal value.

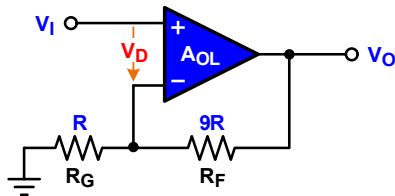


Figure 1. Non-Inverting Amplifier

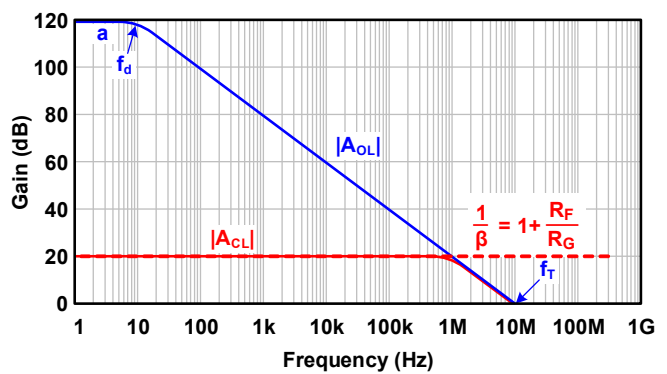


Figure 2. Frequency Responses of A_{OL} and A_{CL}

If A_{OL} were infinite (the ideal case), then A_{CL} would maintain its ideal value across the entire frequency range. This is shown as the dotted line in [Figure 2](#). Unfortunately, A_{CL} never reaches infinity. It has a very high value at DC up to the dominant pole, f_d , but then drops off at a rate of 20dB per decade of frequency, representing a typical low-pass characteristic:

$$(EQ. 2) \quad A_{OL} = \frac{V_O}{V_D} = \frac{a}{1 + j \frac{f}{f_d}}$$

Unfortunately, f_d is usually not specified in datasheets. Instead, the two most commonly specified parameters are the DC open-loop gain, a , and the gain-bandwidth of the op-amp, f_T . During the A_{OL} roll off, the gain-bandwidth is constant and $a \cdot f_d = f_T$. Then, substituting f_d in [Equation 2](#) with $f_d = f_T/a$ gives [Equation 3](#):

$$(EQ. 3) \quad A_{OL} = \frac{a}{1 + j \frac{a \cdot f}{f_T}}$$

This equation describes the frequency response of $|A_{OL}|$ purely based on datasheet parameters.

Because the frequency dependent A_{OL} appears in the k_{Acc} term, this makes the gain accuracy also frequency dependent. Therefore, with decreasing open-loop gain, the gain accuracy also decreases. This decrease can be observed in the Bode plot of [Figure 2](#), which depicts the magnitude functions of A_{OL} and A_{CL} . At the frequency, where $1/\beta$ crosses A_{OL} , the gain accuracy is only 0.707, which represents a -3dB drop of A_{CL} from $1/\beta$. This frequency is also known as the signal bandwidth of the amplifier.

With the gain error being defined as the difference between one and the gain accuracy magnitude shown in [Equation 4](#):

$$(EQ. 4) \quad |E_G| = 1 - |k_{Acc}|$$

the gain error at the -3dB frequency is $E_G = 1 - 0.707 = 0.293$, or 29.3%. Although not clearly visible in the Bode plot of [Figure 2 on page 2](#), E_G gradually increases with frequency.

Inserting [Equation 3](#) into the k_{Acc} term of [Equation 1](#) results in the complex and frequency dependent gain accuracy:

$$k_{Acc} = \frac{1}{1 + \frac{1}{a \cdot \beta} + j \frac{f}{f_T \cdot \beta}}, \text{ whose magnitude function is: } |k_{Acc}| = \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}}$$

Inserting $|k_{Acc}|$ into [Equation 4](#) provides the frequency dependent gain error function with:

$$(EQ. 5) \quad |E_G| = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}}$$

Based on the op-amp datasheet parameters a and f_T and the ideal gain of the amplifier circuit, $1/\beta$, the closed-loop gain error can be calculated for individual frequencies.

1.1 Calculation Examples

[Figures 1](#) and [2](#) depict the DC open-loop gain and gain bandwidth of the op-amp with $a = 120\text{dB}$ (10^6 V/V) and $f_T = 10\text{MHz}$ respectively, and the ideal closed-loop gain of the non-inverting amplifier with $1/\beta = 10\text{V/V}$.

You can assume that this wide-band op-amp would amplify a 500kHz input sinewave with a relatively small gain error of less than 1%. However, applying [Equation 5](#) results in a rather high gain error of 10.6%.

$$E_G = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}} = 1 - \frac{1}{\sqrt{\left(1 + \frac{10 \text{ V/V}}{10^6 \text{ V/V}}\right)^2 + \left(\frac{500\text{kHz} \cdot 10 \text{ V/V}}{10\text{MHz}}\right)^2}} = 0.106 \quad \text{or} \quad 10.6\%$$

To determine the maximum signal frequency, f_{max} , this op-amp can amplify with a gain error of $E_G \leq 0.1\%$, [Equation 5](#) is solved for f and the index *max* added:

$$(EQ. 6) \quad f_{max} \leq f_T \cdot \beta \cdot \sqrt{\left(\frac{1}{1 - E_{G_{max}}}\right)^2 - \left(1 + \frac{1}{a \cdot \beta}\right)^2}$$

Inserting the desired gain error limit then yields a maximum signal frequency of:

$$f_{max} \leq \frac{10\text{MHz}}{10 \text{ V/V}} \sqrt{\left(\frac{1}{1 - 0.001}\right)^2 - \left(1 + \frac{10 \text{ V/V}}{10^6 \text{ V/V}}\right)^2} = 44.5\text{kHz}$$

The maximum signal frequency is an astonishingly narrow frequency band that can be amplified with less than 0.1% gain error.

To find the minimum gain bandwidth, $f_{T\text{-min}}$, of an op-amp that amplifies the 500kHz sinewave with maximum 0.1% gain error, [Equation 5](#) is solved for f_T and the index min added:

$$(EQ. 7) \quad f_{T\text{min}} \geq \frac{1}{\beta} \cdot \frac{f}{\sqrt{\left(\frac{1}{1-E_{G\text{max}}}\right)^2 - \left(1 + \frac{1}{a \cdot \beta}\right)^2}}$$

Inserting the desired gain error and signal frequency limits requires a minimum gain bandwidth of:

$$f_{T\text{min}} \geq \frac{10\text{ V/V} \cdot 500\text{kHz}}{\sqrt{\left(\frac{1}{1-10^{-3}}\right)^2 - \left(1 + \frac{10\text{ V/V}}{10^6\text{ V/V}}\right)^2}} = 112\text{MHz}$$

The minimum gain bandwidth is an impressively high gain bandwidth, needed to amplify frequencies up to 500kHz with less than 0.1% gain error.

When amplifying a wide-band signal, some designers consider the use of Current Feedback (CFB) op-amps. While these amplifiers provide the necessary bandwidth, they lack however in DC-precision, such as DC open-loop gain.

2. Closed-Loop Gain Error as a Function of Open-Loop Gain for the Inverting Amplifier

The closed-loop gain of the inverting amplifier in [Figure 3](#) is:

$$(EQ. 8) \quad A_{CL} = -\left(\frac{1}{\beta} - 1\right) \cdot \frac{1}{1 + \frac{1}{A_{OL}\beta}} = -\left(\frac{1}{\beta} - 1\right) \cdot k_{Acc}$$

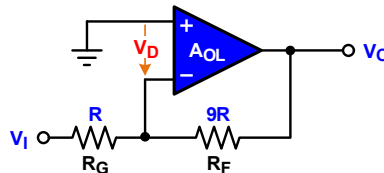


Figure 3. Inverting Amplifier

[Equation 8](#) shows that the ideal closed loop gain is 1V/V less than that of the non-inverting amplifier, the gain accuracy however, is the same.

Note: For the same feedback factor, the gain errors of the inverting and non-inverting amplifier configurations are identical.

3. Closed-Loop Gain Error of the Non-Inverting and Inverting Amplifier for equal Closed-Loop Gain

Matching the DC gain of an inverting amplifier with that of a non-inverting amplifier, requires the increase of the R_F value in the inverting amplifier's feedback path, so that:

$$\frac{R_{F(\text{inv})}}{R_G} = 1 + \frac{R_{F(\text{nin})}}{R_G}$$

With the indices (inv) and (nin) indicating the inverting and non-inverting configurations. Solving for $R_{F(\text{inv})}$ gives: $R_{F(\text{inv})} = R_{F(\text{nin})} + R_G$. This of course changes the feedback factor of the non-inverting amplifier. Comparing both feedback factors shows:

$$\beta_{\text{nin}} = \frac{R_G}{R_{F(\text{nin})} + R_G} \text{ for the non-inverting and } \beta_{\text{inv}} = \frac{R_G}{R_{F(\text{nin})} + 2R_G} \text{ for the inverting configuration.}$$

Inserting these feedback factors into the gain error equations shows their difference in gain accuracy and hence, gain error:

$$|E_{G(\text{nin})}| = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a} \cdot \frac{1}{\beta_{\text{nin}}}\right)^2 + \left(\frac{f}{f_T} \cdot \frac{1}{\beta_{\text{nin}}}\right)^2}}$$

and

$$|E_{G(\text{inv})}| = 1 - \frac{1}{\sqrt{\left[1 + \frac{1}{a} \cdot \left(\frac{1}{\beta_{\text{nin}}} + 1\right)\right]^2 + \left[\frac{f}{f_T} \left(\frac{1}{\beta_{\text{nin}}} + 1\right)\right]^2}}$$

For the inverting amplifier, the denominator of the gain accuracy term increases; therefore, lowering the gain accuracy and in turn, increasing the gain error, when compared to the non-inverting case.

Note: For equal closed-loop gain, the gain error of the inverting amplifier is larger than that of the non-inverting amplifier.

4. Conclusion

Precision amplifiers allow for closed-loop gain errors in the range of 10^{-4} to 10^{-3} V/V, or 0.01 to 0.1%. This gain error is due to the open-loop gain roll off of the op-amp and occurs far below the -3dB frequency of the amplifier's closed-loop gain. [Figure 4](#) shows a comparison between the actual closed-loop gain, A_{CL} , and the ideal closed loop gain, A_{CLi} .

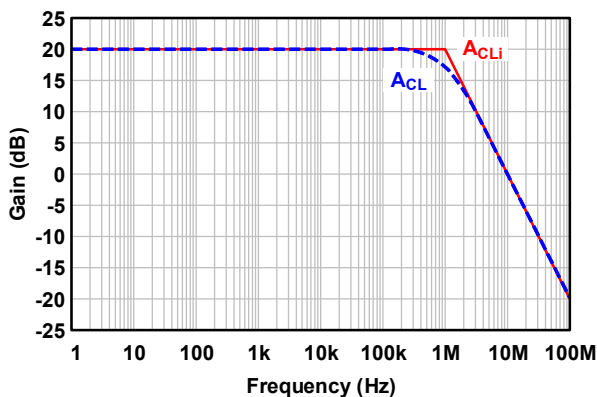


Figure 4. Ideal versus Actual Closed-Loop Gain

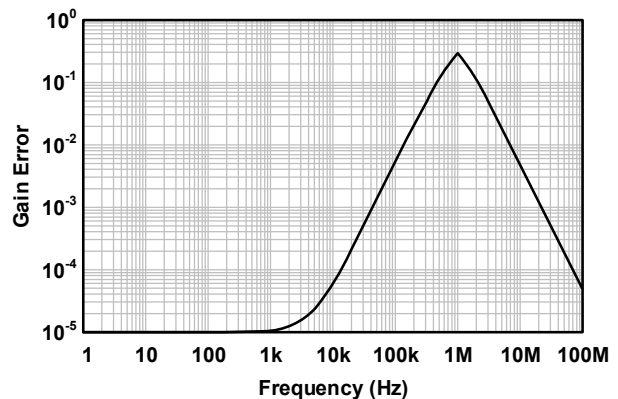


Figure 5. Closed-Loop Gain Error over Frequency

It becomes obvious that small gain errors cannot be identified from a Bode plot. However, this application note has shown that significant gain errors affecting precision performance already occur far below the -3dB frequency

of the amplifier's closed-loop gain ([Figure 5 on page 5](#)). Therefore, to minimize gain error over a wide signal bandwidth, the use of op-amps with high gain-bandwidth is required.

With regards to the size of the gain error for different amplifier configurations, it showed the following:

- For equal feedback factors, the gain errors of the inverting and noninverting configurations are the same.
- For equal closed-loop gains, the gain error of the inverting amplifier is larger than that of the noninverting amplifier.

5. Revision History

Rev.	Date	Description
1.00	Aug.15.19	Initial release

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(Rev.4.0-1 November 2017)

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