Operational Amplifiers

Closed-Loop Gain Error

Abstract

Engineers new to operational amplifier (op-amp) circuit design often attribute amplifier gain error purely to resistor tolerances, while ignoring the gain error caused by the op-amp. This can prove detrimental, as the gain error of the op-amp can be higher than the one due to resistor tolerances. Also, op-amp gain error is frequency dependent, which can cause signal distortions when amplifying input signals containing multiple frequency components.

This application note explains the impact of op-amp open-loop gain on the closed-loop gain error and derives equations for the following:

- Calculating the gain error for a given op-amp at a specific frequency
- Identifying the maximum signal frequency that can be amplified within a specified gain error
- Determining the minimum gain bandwidth of an op-amp, required to amplify a given signal frequency within a specified gain error.

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1. Closed-Loop Gain Error as a Function of Open-Loop Gain for the Non-Inverting Amplifier

The gain error is defined as the deviation of the actual closed-loop gain of the amplifier from its ideal value. For example, the closed-loop gain, A_{CL}, or transfer function, V_O/V_I, of the non-inverting amplifier shown in Figure 1 is:

(EQ. 1) $$A_{CL} = \frac{1}{\beta} \cdot k_{Acc} \quad \text{with} \quad \frac{1}{\beta} = 1 + \frac{R_F}{R_G} \quad \text{and} \quad k_{Acc} = \frac{1}{1 + \frac{1}{A_{OL}\beta}}$$

Here, β is the feedback factor of the amplifier, and 1/β is the ideal closed-loop gain of the non-inverting amplifier for $A_{OL} = \infty$. The second factor, k_{Acc}, is the gain accuracy, which determines how close the actual closed-loop gain approaches the ideal value.

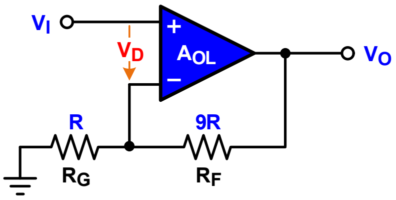

Figure 1. Non-Inverting Amplifier

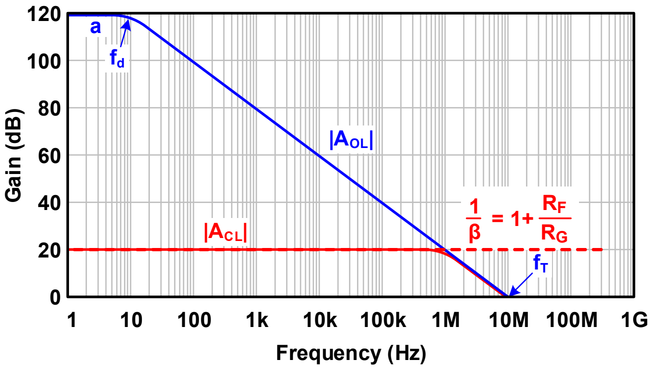

Figure 2. Frequency Responses of A_{OL} and A_{CL}

If A_{OL} were infinite (the ideal case), then A_{CL} would maintain its ideal value across the entire frequency range. This is shown as the dotted line in Figure 2. Unfortunately, A_{CL} never reaches infinity. It has a very high value at DC up to the dominant pole, f_d, but then drops off at a rate of 20dB per decade of frequency, representing a typical low-pass characteristic:

(EQ. 2) $$A_{OL} = \frac{V_O}{V_D} = \frac{a}{1 + j\frac{f}{f_d}}$$

Unfortunately, f_d is usually not specified in datasheets. Instead, the two most commonly specified parameters are the DC open-loop gain, a, and the gain-bandwidth of the op-amp, f_T. During the A_{OL} roll off, the gain-bandwidth is constant and $a \cdot f_d = f_T$. Then, substituting f_d in Equation 2 with $f_d = f_T/a$ gives Equation 3:

(EQ. 3) $$A_{OL} = \frac{a}{1 + j\frac{a \cdot f}{f_T}}$$

This equation describes the frequency response of $|A_{OL}|$ purely based on datasheet parameters.

Because the frequency dependent A_{OL} appears in the k_{Acc} term, this makes the gain accuracy also frequency dependent. Therefore, with decreasing open-loop gain, the gain accuracy also decreases. This decrease can be observed in the Bode plot of Figure 2, which depicts the magnitude functions of A_{OL} and A_{CL}. At the frequency, where 1/β crosses A_{OL}, the gain accuracy is only 0.707, which represents a -3dB drop of A_{CL} from 1/β. This frequency is also known as the signal bandwidth of the amplifier.

With the gain error being defined as the difference between one and the gain accuracy magnitude shown in Equation 4:

(EQ. 4)
$$|E_G| = 1 - |k_{Acc}|$$

the gain error at the -3dB frequency is $E_G = 1 - 0.707 = 0.293$, or 29.3%. Although not clearly visible in the Bode plot of Figure 2 on page 2, E_G gradually increases with frequency.

Inserting Equation 3 into the k_{Acc} term of Equation 1 results in the complex and frequency dependent gain accuracy:

$$k_{Acc} = \frac{1}{1 + \frac{1}{a \cdot \beta} + j\frac{f}{f_T \cdot \beta}}$$, whose magnitude function is: $$|k_{Acc}| = \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}}$$

Inserting $|k_{Acc}|$ into Equation 4 provides the frequency dependent gain error function with:

(EQ. 5)
$$|E_G| = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}}$$

Based on the op-amp datasheet parameters *a* and f_T and the ideal gain of the amplifier circuit, 1/β, the closed-loop gain error can be calculated for individual frequencies.

1.1 Calculation Examples

Figures 1 and 2 depict the DC open-loop gain and gain bandwidth of the op-amp with a = 120dB (10^6 V/V) and f_T = 10MHz respectively, and the ideal closed-loop gain of the non-inverting amplifier with 1/β = 10V/V.

You can assume that this wide-band op-amp would amplify a 500kHz input sinewave with a relatively small gain error of less than 1%. However, applying Equation 5 results in a rather high gain error of 10.6%.

$$E_G = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a \cdot \beta}\right)^2 + \left(\frac{f}{f_T \cdot \beta}\right)^2}} = 1 - \frac{1}{\sqrt{\left(1 + \frac{10\,V/V}{10^6\,V/V}\right)^2 + \left(\frac{500kHz \cdot 10\,V/V}{10MHz}\right)^2}} = 0.106 \quad \text{or} \quad 10.6\%$$

To determine the maximum signal frequency, f_{max}, this op-amp can amplify with a gain error of $E_G \leq 0.1\%$, Equation 5 is solved for f and the index *max* added:

(EQ. 6)
$$f_{max} \leq f_T \cdot \beta \cdot \sqrt{\left(\frac{1}{1 - E_{G\,max}}\right)^2 - \left(1 + \frac{1}{a \cdot \beta}\right)^2}$$

Inserting the desired gain error limit then yields a maximum signal frequency of:

$$f_{max} \leq \frac{10MHz}{10\,V/V}\sqrt{\left(\frac{1}{1 - 0.001}\right)^2 - \left(1 + \frac{10\,V/V}{10^6\,V/V}\right)^2} = 44.5kHz$$

The maximum signal frequency is an astonishingly narrow frequency band that can be amplified with less than 0.1% gain error.

To find the minimum gain bandwidth, $f_{T\text{-}min}$, of an op-amp that amplifies the 500kHz sinewave with maximum 0.1% gain error, Equation 5 is solved for f_T and the index min added:

(EQ. 7)
$$f_{T\,min} \geq \frac{1}{\beta} \cdot \frac{f}{\sqrt{\left(\frac{1}{1-E_{G\,max}}\right)^2 - \left(1+\frac{1}{a\cdot\beta}\right)^2}}$$

Inserting the desired gain error and signal frequency limits requires a minimum gain bandwidth of:

$$f_{T\,min} \geq \frac{10\,V/V \cdot 500kHz}{\sqrt{\left(\frac{1}{1-10^{-3}}\right)^2 - \left(1+\frac{10\,V/V}{10^6\,V/V}\right)^2}} = 112MHz$$

The minimum gain bandwidth is an impressively high gain bandwidth, needed to amplify frequencies up to 500kHz with less than 0.1% gain error.

When amplifying a wide-band signal, some designers consider the use of Current Feedback (CFB) op-amps. While these amplifiers provide the necessary bandwidth, they lack however in DC-precision, such as DC open-loop gain.

2. Closed-Loop Gain Error as a Function of Open-Loop Gain for the Inverting Amplifier

The closed-loop gain of the inverting amplifier in Figure 3 is:

(EQ. 8)
$$A_{CL} = -\left(\frac{1}{\beta}-1\right) \cdot \frac{1}{1+\frac{1}{A_{OL}\beta}} = -\left(\frac{1}{\beta}-1\right) \cdot k_{Acc}$$

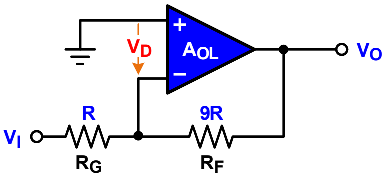

Figure 3. Inverting Amplifier

Equation 8 shows that the ideal closed loop gain is 1V/V less than that of the non-inverting amplifier, the gain accuracy however, is the same.

Note: For the same feedback factor, the gain errors of the inverting and non-inverting amplifier configurations are identical.

3. Closed-Loop Gain Error of the Non-Inverting and Inverting Amplifier for equal Closed-Loop Gain

Matching the DC gain of an inverting amplifier with that of a non-inverting amplifier, requires the increase of the R_F value in the inverting amplifier's feedback path, so that:

$$\frac{R_{F(inv)}}{R_G} = 1 + \frac{R_{F(nin)}}{R_G}$$

With the indices (inv) and (nin) indicating the inverting and non-inverting configurations. Solving for $R_{F(inv)}$ gives: $R_{F(inv)} = R_{F(nin)} + R_G$. This of course changes the feedback factor of the non-inverting amplifier. Comparing both feedback factors shows:

$\beta_{nin} = \dfrac{R_G}{R_{F(nin)} + R_G}$ for the non-inverting and $\beta_{inv} = \dfrac{R_G}{R_{F(nin)} + 2R_G}$ for the inverting configuration.

Inserting these feedback factors into the gain error equations shows their difference in gain accuracy and hence, gain error:

$$\left|E_{G(nin)}\right| = 1 - \frac{1}{\sqrt{\left(1 + \frac{1}{a} \cdot \frac{1}{\beta_{nin}}\right)^2 + \left(\frac{f}{f_T} \cdot \frac{1}{\beta_{nin}}\right)^2}}$$

and

$$\left|E_{G(inv)}\right| = 1 - \frac{1}{\sqrt{\left[1 + \frac{1}{a} \cdot \left(\frac{1}{\beta_{nin}} + 1\right)\right]^2 + \left[\frac{f}{f_T}\left(\frac{1}{\beta_{nin}} + 1\right)\right]^2}}$$

For the inverting amplifier, the denominator of the gain accuracy term increases; therefore, lowering the gain accuracy and in turn, increasing the gain error, when compared to the non-inverting case.

Note: For equal closed-loop gain, the gain error of the inverting amplifier is larger than that of the non-inverting amplifier.

4. Conclusion

Precision amplifiers allow for closed-loop gain errors in the range of 10^{-4} to 10^{-3} V/V, or 0.01 to 0.1%. This gain error is due to the open-loop gain roll off of the op-amp and occurs far below the -3dB frequency of the amplifier's closed-loop gain. Figure 4 shows a comparison between the actual closed-loop gain, A_{CL}, and the ideal closed loop gain, A_{CLi}.

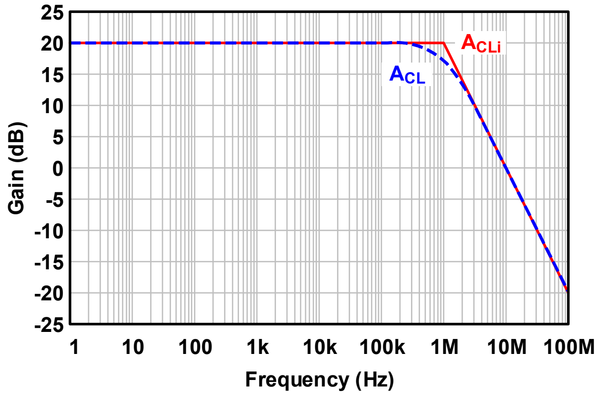

Figure 4. Ideal versus Actual Closed-Loop Gain

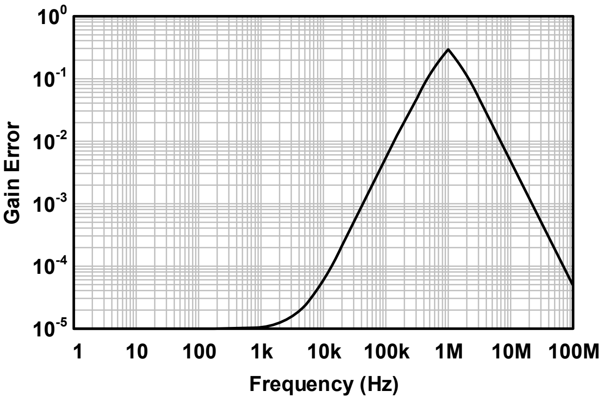

Figure 5. Closed-Loop Gain Error over Frequency

It becomes obvious that small gain errors cannot be identified from a Bode plot. However, this application note has shown that significant gain errors affecting precision performance already occur far below the -3dB frequency of the amplifier's closed-loop gain (Figure 5 on page 5). Therefore, to minimize gain error over a wide signal bandwidth, the use of op-amps with high gain-bandwidth is required.

With regards to the size of the gain error for different amplifier configurations, it showed the following:

- For equal feedback factors, the gain errors of the inverting and noninverting configurations are the same.
- For equal closed-loop gains, the gain error of the inverting amplifier is larger than that of the noninverting amplifier.

5. Revision History

Rev.	Date	Description
1.00	Aug.15.19	Initial release

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