

# Motor Control Application

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## Vector Control of Three Phase Induction Motor (Algorithms)

Sep 05, 2014

### Introduction

This application note describes the three phase induction motor vector control algorithms used in a sample program for Renesas microcontrollers.

### Contents

1. Overview .....	2
2. Three-Phase Induction Motor Voltage Equations .....	2
3. Indirect Vector Control .....	7

### 1. Overview

This application note describes the three phase induction motor vector control algorithms used in a sample program for Renesas microcontrollers.

### 2. Voltage Equations of Three Phase Induction Motor

#### 2.1 Control Model of Three Phase Induction Motor

The voltage equations for three phase induction motors can be expressed as shown below.

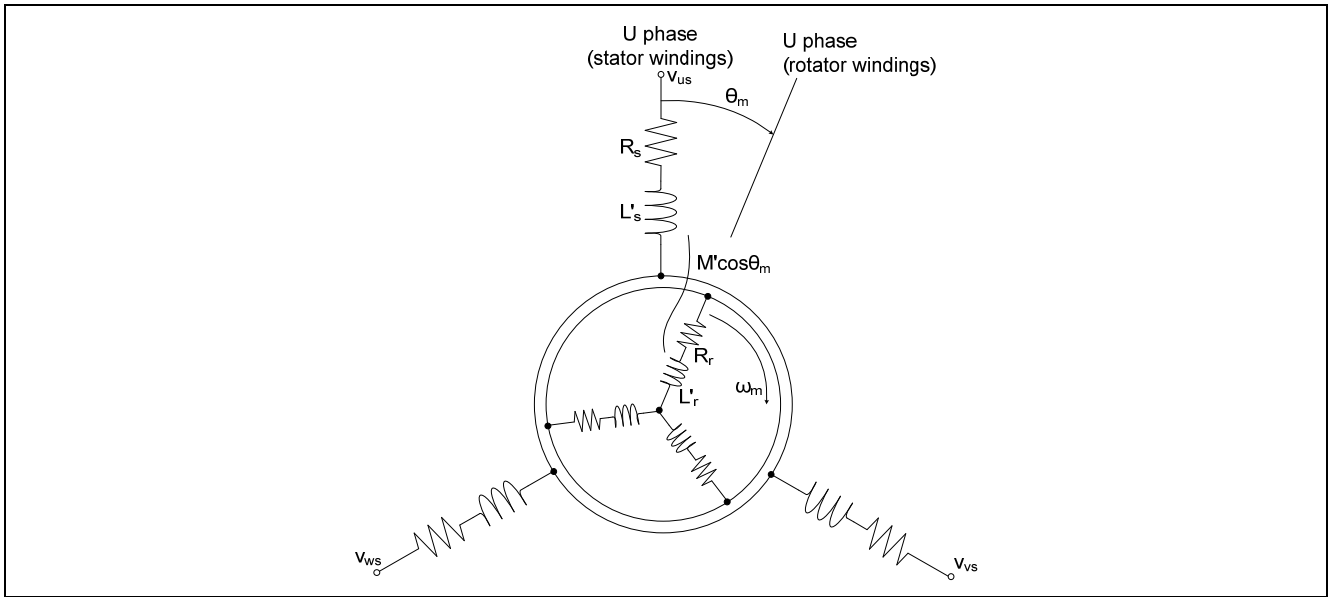


Figure 2.1 Overview of Three Phase Induction Motor

$$\begin{pmatrix} v_{us} \\ v_{vs} \\ v_{ws} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_s + PL'_s & -P \frac{M'}{2} & -P \frac{M'}{2} & PM' \cos \theta_m & PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) & PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) \\ -P \frac{M'}{2} & R_s + PL'_s & -P \frac{M'}{2} & PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) & PM' \cos \theta_m & PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) \\ -P \frac{M'}{2} & -P \frac{M'}{2} & R_s + PL'_s & PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) & PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) & PM' \cos \theta_m \\ PM' \cos \theta_m & PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) & PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) & R_r + PL'_r & -P \frac{M'}{2} & -P \frac{M'}{2} \\ PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) & PM' \cos \theta_m & PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) & -P \frac{M'}{2} & R_r + PL'_r & -P \frac{M'}{2} \\ PM' \cos \left( \theta_m - \frac{2\pi}{3} \right) & PM' \cos \left( \theta_m + \frac{2\pi}{3} \right) & PM' \cos \theta_m & -P \frac{M'}{2} & -P \frac{M'}{2} & R_r + PL'_r \end{pmatrix} \begin{pmatrix} i_{us} \\ i_{vs} \\ i_{ws} \\ i_{ur} \\ i_{vr} \\ i_{wr} \end{pmatrix} \dots (1)$$

$$L'_s = l_s + M', \quad L'_r = l_r + M'$$

$v_u, v_v, v_w$  : Stator voltages for each phase

$\theta_m$  : Angle from the U phase stator windings to the U phase rotator windings

$i_{us}, i_{vs}, i_{ws}$  : Stator currents for each phase

$L_s'$  : Stator winding self inductance

$i_{ur}, i_{vr}, i_{wr}$  : Rotor currents for each phase

$L_r'$  : Rotor winding self inductance

$R_s$  : Stator winding resistance

$l_s$  : Stator winding leakage inductance

$R_r$  : Rotor winding resistance

$l_r$  : Rotor winding leakage inductance

$P$  : Differential operator

$M'$  : Mutual inductance between windings

## 2.2 Motor Control System Voltage Equations

### 2.2.1 $\alpha\beta$ Transformation

Here, a coordinate transformation is performed on the voltage equations for the three phases to express them as two phases DC.

First, to express these as two phases DC, the following transformation matrix is used to perform an  $\alpha\beta$  transformation.

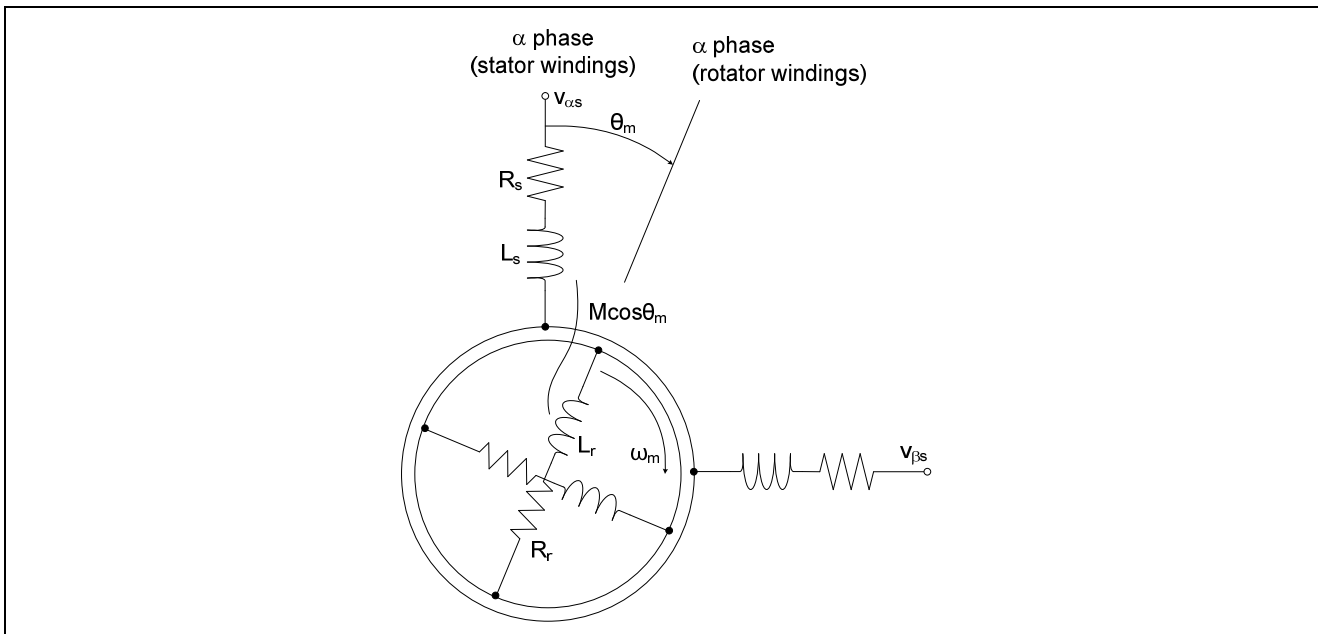


Figure 2.2  $\alpha\beta$  Transformation Overview

$$C = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \dots(2)$$

The  $\alpha\beta$  transformation allows the three phase induction motor voltage equations to be expressed in the  $\alpha\beta$  coordinate system as shown below.

$$\begin{pmatrix} v_{\alpha s} \\ v_{\beta s} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_s + PL_s & 0 & PM \cos \theta_m & -PM \sin \theta_m \\ 0 & R_s + PL_s & PM \sin \theta_m & PM \cos \theta_m \\ PM \cos \theta_m & PM \sin \theta_m & R_r + PL_r & 0 \\ -PM \sin \theta_m & PM \cos \theta_m & 0 & R_r + PL_r \end{pmatrix} \begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{\alpha r} \\ i_{\beta r} \end{pmatrix} \dots (3)$$

$$L_s = l_s + M, \quad L_r = l_r + M, \quad M = \frac{3}{2} M'$$

$v_{\alpha}, v_{\beta}$  : Stator voltages for each phase       $L_s$  : Stator winding inductance

$i_{\alpha s}, i_{\beta s}$  : Stator currents for each phase       $L_r$  : Rotor winding inductance

$i_{\alpha r}, i_{\beta r}$  : Rotor currents for each phase       $M$  : Mutual inductance between windings

### 2.2.2 dq Transformation

A dq transformation is performed and the following transformation matrix is used to express the result in a fixed orthogonal coordinate system. The d axis can be taken to be anywhere, but here the u phase ( $\alpha$  phase) is used.

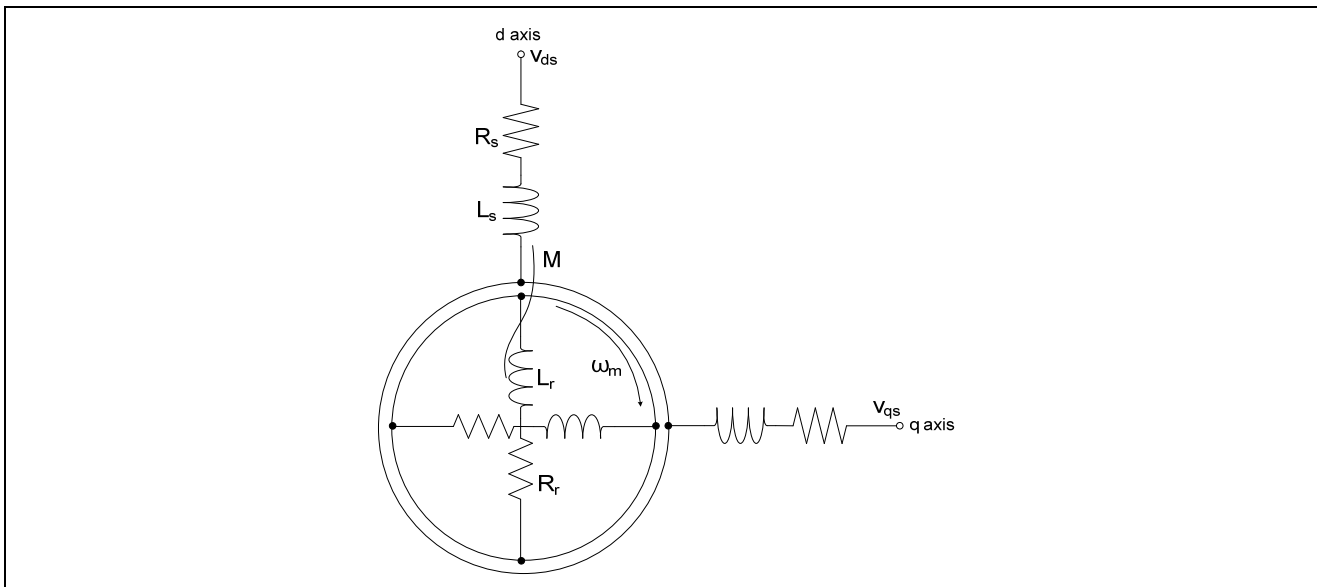


Figure 2.3 dq Transformation Overview

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_m & -\sin\theta_m \\ 0 & 0 & \sin\theta_m & \cos\theta_m \end{pmatrix} \dots (4)$$

The dq transformation allows the three phase induction motor voltage equations to be expressed in the dq coordinate system as shown below.

$$\begin{pmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_s + PL_s & 0 & PM & 0 \\ 0 & R_s + PL_s & 0 & PM \\ PM & \omega_m M & R_r + PL_r & \omega_m L_r \\ -\omega_m M & PM & -\omega_m L_r & R_r + PL_r \end{pmatrix} \begin{pmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{pmatrix} \dots (5)$$

$v_{ds}, v_{qs}$  : Stator voltages for each phase  
 $i_{ds}, i_{qs}$  : Stator currents for each phase  
 $i_{dr}, i_{qr}$  : Rotor currents for each phase  
 $\omega_m$  : Stator angular speed

2.2.3  $\gamma\delta$  Transformation

A  $\gamma\delta$  transformation is performed and the following transformation matrix is used to express the result as two phases DC.

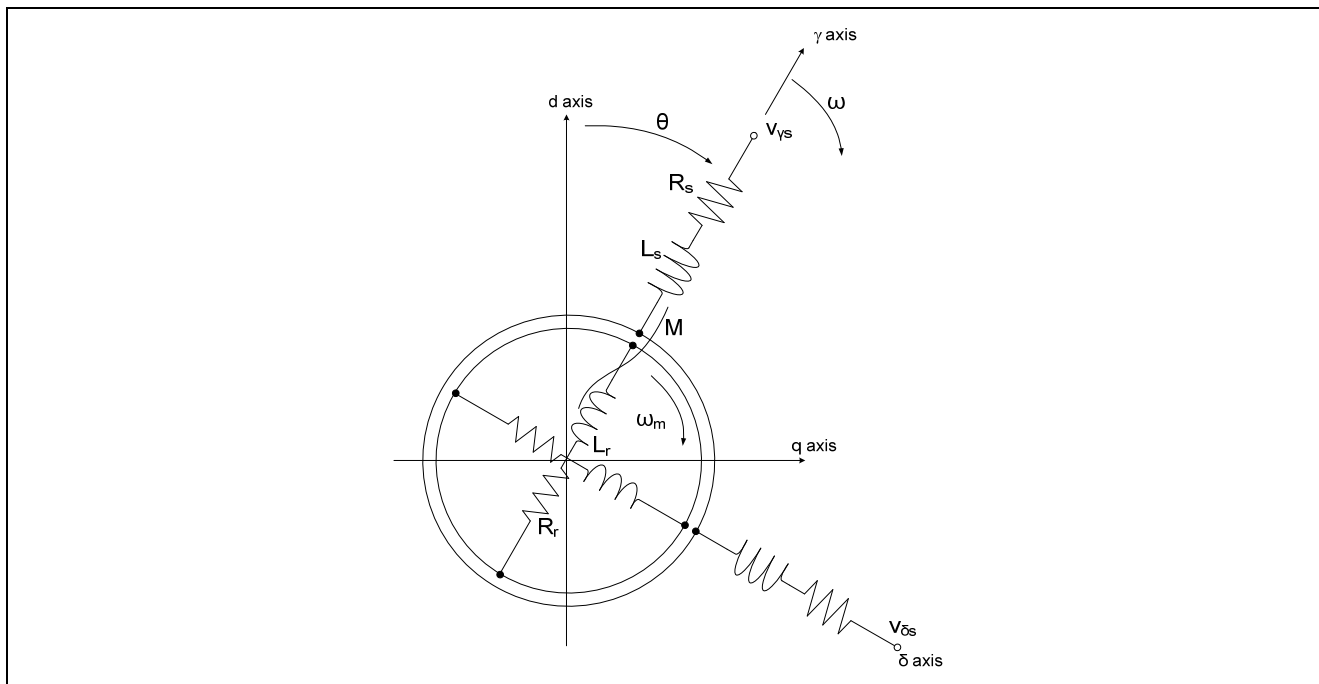


Figure 2.4  $\gamma\delta$  Transformation Overview

$$C = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{pmatrix} \quad \dots (6)$$

$\theta$  : Angle from the d axis to the  $\gamma$  axis.

The  $\gamma\delta$  transformation allows the three phases induction motor voltage equations to be expressed in the  $\gamma\delta$  coordinate system as shown below.

$$\begin{pmatrix} v_{\gamma s} \\ v_{\delta s} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_s + PL_s & -\omega L_s & PM & -\omega M \\ \omega L_s & R_s + PL_s & \omega M & PM \\ PM & -(\omega - \omega_m)M & R_r + PL_r & -(\omega - \omega_m)L_r \\ (\omega - \omega_m)M & PM & (\omega - \omega_m)L_r & R_r + PL_r \end{pmatrix} \begin{pmatrix} i_{\gamma s} \\ i_{\delta s} \\ i_{\gamma r} \\ i_{\delta r} \end{pmatrix} \quad \dots (7)$$

$v_{\gamma s}, v_{\delta s}$  : Stator voltages for each phase       $i_{\gamma r}, i_{\delta r}$  : Rotor currents for each phase

$i_{\gamma s}, i_{\delta s}$  : Stator currents for each phase       $\omega$  : Angular frequency of the current

As shown above, the three phase currents flowing in the stator and rotor can be expressed as two phases DC by viewing them from a coordinate system that is rotating at angular speed  $\omega$ .

**2.2.4 Expressions Based on the Rotor Interlinkage Flux**

When the rotor interlinkage flux is used instead of the rotor current in equation (7), the equations become as follows.

$$\begin{pmatrix} v_{\gamma s} \\ v_{\delta s} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_s + PL_\sigma & -\omega L_\sigma & P \frac{M}{L_r} & -\omega \frac{M}{L_r} \\ \omega L_\sigma & R_s + PL_\sigma & \omega \frac{M}{L_r} & P \frac{M}{L_r} \\ -R_r \frac{M}{L_r} & 0 & \frac{R_r}{L_r} + P & -\omega_s \\ 0 & -R_r \frac{M}{L_r} & \omega_s & \frac{R_r}{L_r} + P \end{pmatrix} \begin{pmatrix} i_{\gamma s} \\ i_{\delta s} \\ \phi_{\gamma r} \\ \phi_{\delta r} \end{pmatrix} \quad \dots (8)$$

$$\phi_{\gamma r} = Mi_{\gamma s} + L_r i_{\gamma r}, \quad \phi_{\delta r} = Mi_{\delta s} + L_r i_{\delta r}, \quad L_\sigma = L_s - \frac{M^2}{L_r}, \quad \omega_s = \omega - \omega_m$$

$\phi_{\gamma r}, \phi_{\delta r}$  : Rotor interlinkage flux for each phase       $\omega_s$  : Slip frequency

**2.2.5 Torque**

The magnitude of the torque can be expressed as shown below by the rotor current and rotor flux due to the stator current.

$$T = P_n \frac{M}{L_r} (\phi_{\gamma r} i_{\delta s} - \phi_{\delta r} i_{\gamma s}) \quad \dots (9)$$

$T$  : Motor torque       $P_n$  : Number of poles

### 3. Indirect Vector Control

#### 3.1 Vector Control of Three Phase Induction Motor

Vector control is a method in which the flux and generated torque are controlled by independently currents that flow perpendicularly to the flux and current that generates the rotor flux. When we take as a premise that the rotor flux will be controlled so that it is held constant, it is necessary to detect the rotor flux. Here, there are two techniques: direct vector control, in which a flux sensor is directly attached to detect the flux, and indirect vector control (which is used in this sample program) in which the flux is calculated from the detected values of the terminal voltages, currents, and rotational speed.

#### 3.2 Principles of Indirect Vector Control

In indirect vector control, the torque and rotor interlinkage flux are taken to be the objects of control, and current control is substituted for the control of the two parameters.

When the torque can be determined from equation (9) as shown below when the rotor interlinkage flux is defined so that it matches the  $\gamma$  axis.

$$\phi_{\gamma r} = \phi_r \quad \phi_{\delta r} = 0 \quad \dots (10)$$

$$T = P_n \frac{M}{L_r} \phi_r i_{\delta s} \quad \dots (11)$$

Also, the rotor interlinkage flux can be determined from equation (8) as shown below.

$$\phi_r = \frac{M}{1 + PT_r} i_{\gamma s} \quad \dots (12)$$

$$T_r = \frac{L_r}{R_r} : \text{Rotor time constant}$$

Using the above, the torque and the rotor interlinkage flux can be controlled by controlling the stator current.

Furthermore, the slip angular frequency can be determined from equations (8) and (12) as shown below.

$$\omega_s = \frac{M}{T_r \phi_r} i_{\delta s} = \frac{i_{\delta s}}{T_r \left( \frac{1}{1 + PT_r} \right) i_{\gamma s}} \quad \dots (13)$$

The  $\gamma$  axis angle, which is the direction of the rotor interlinkage flux, can be determined by integrating the sum of the slip angular frequency and the rotor angular frequency.

This method is called indirect vector control since the  $\gamma$  axis angle is determined indirectly from the slip frequency. It is also called slip frequency control.

### 3.3 Speed Sensorless Vector Control

When vector control is used without a speed sensor, it is necessary to infer the speed by some method. While there are a variety of methods for inferring the speed, this application note presents a method based on voltage control.

If the rotor interlinkage flux is fixed, the slip angular frequency has a proportional relationship with the torque current as shown in equation (13). This means that vector control is possible by controlling the stator angular frequency so that the torque value and that control value match.

$$\omega_s = \frac{i_{\delta s}}{T_r \left( \frac{1}{1 + PT_r} \right) i_{\gamma s}} = K_s i_{\delta s} \quad \dots (14)$$

$K_s$  : Constant

The rotor angular frequency can be determined by subtracting the slip angular frequency from the stator angular frequency as shown below.

$$\omega_m = \omega - \omega_s \quad \dots (15)$$

The conversion from control current to control voltage is performed using equation (8) as shown below.

$$\begin{aligned} v_{\gamma s} &= R_s i_{\gamma s} - \omega L_{\sigma} i_{\delta s} \\ v_{\delta s} &= R_s i_{\delta s} + \omega L_s i_{\gamma s} \end{aligned} \quad \dots (16)$$

If the detected torque current and command current do not match when this voltage is applied, it means that the  $\gamma$  axis does not match the rotor interlinkage flux direction. Here, the  $\gamma$  axis phase can be taken to be the integral over time of the result of PI amplification of that error.

The figure below shows the block diagram for the flow of control in this control method.

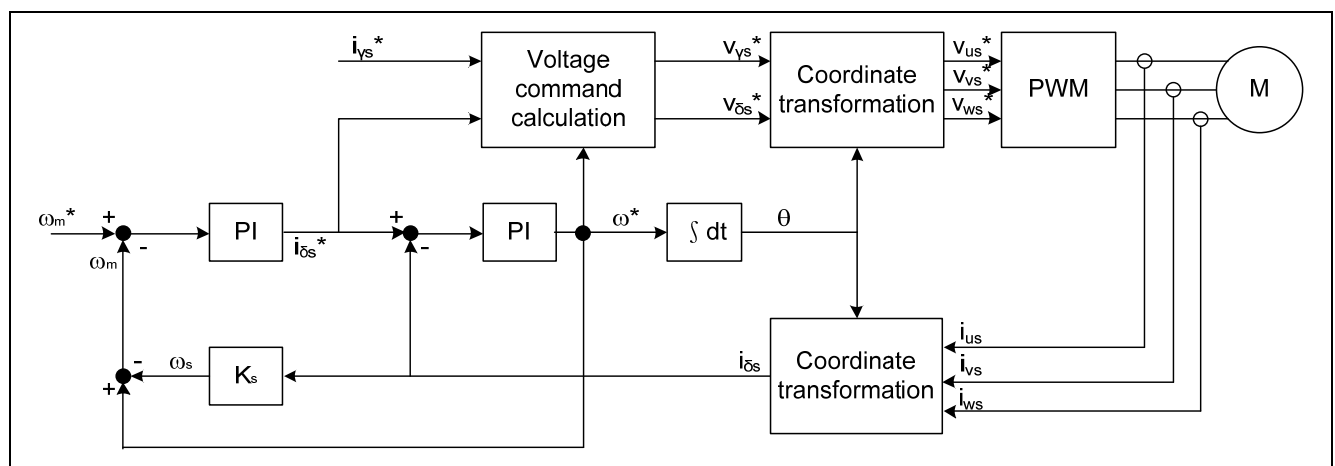


Figure 3.1 Speed Sensorless Vector Control Flowchart



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## Revision History

Rev.	Date	Description	
		Page	Summary
1.00	Sep 05, 2014	—	First edition issued

## General Precautions in the Handling of MPU/MCU Products

The following usage notes are applicable to all MPU/MCU products from Renesas. For detailed usage notes on the products covered by this document, refer to the relevant sections of the document as well as any technical updates that have been issued for the products.

### 1. Handling of Unused Pins

Handle unused pins in accord with the directions given under Handling of Unused Pins in the manual.

- The input pins of CMOS products are generally in the high-impedance state. In operation with an unused pin in the open-circuit state, extra electromagnetic noise is induced in the vicinity of LSI, an associated shoot-through current flows internally, and malfunctions occur due to the false recognition of the pin state as an input signal become possible. Unused pins should be handled as described under Handling of Unused Pins in the manual.

### 2. Processing at Power-on

The state of the product is undefined at the moment when power is supplied.

- The states of internal circuits in the LSI are indeterminate and the states of register settings and pins are undefined at the moment when power is supplied.

In a finished product where the reset signal is applied to the external reset pin, the states of pins are not guaranteed from the moment when power is supplied until the reset process is completed.

In a similar way, the states of pins in a product that is reset by an on-chip power-on reset function are not guaranteed from the moment when power is supplied until the power reaches the level at which resetting has been specified.

### 3. Prohibition of Access to Reserved Addresses

Access to reserved addresses is prohibited.

- The reserved addresses are provided for the possible future expansion of functions. Do not access these addresses; the correct operation of LSI is not guaranteed if they are accessed.

### 4. Clock Signals

After applying a reset, only release the reset line after the operating clock signal has become stable.

When switching the clock signal during program execution, wait until the target clock signal has stabilized.

- When the clock signal is generated with an external resonator (or from an external oscillator) during a reset, ensure that the reset line is only released after full stabilization of the clock signal. Moreover, when switching to a clock signal produced with an external resonator (or by an external oscillator) while program execution is in progress, wait until the target clock signal is stable.

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